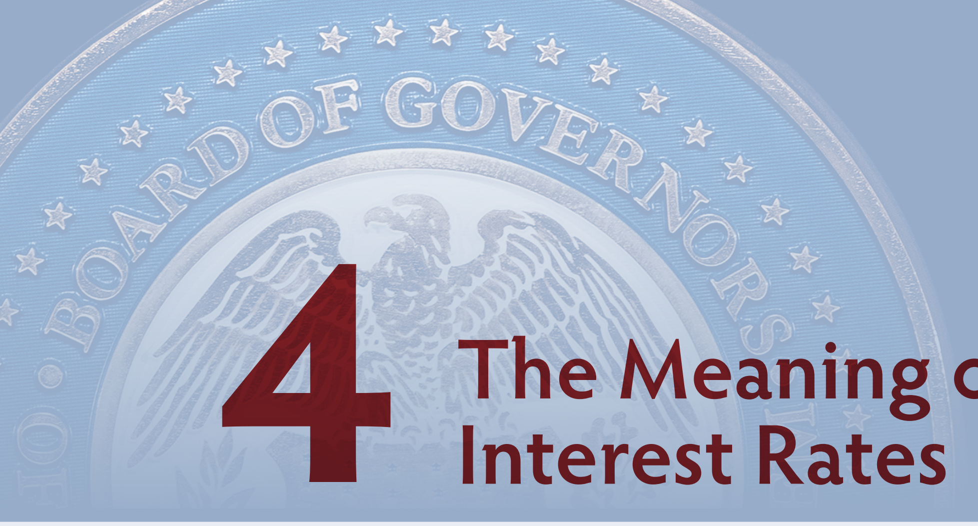


4 The Meaning of Interest Rates

Learning Objectives

- Calculate the present value of future cash flows and the yield to maturity on the four types of credit market instruments.
- Recognize the distinctions among yield to maturity, current yield, rate of return, and rate of capital gain.
- Interpret the distinction between real and nominal interest rates.

Preview

Interest rates are among the most closely watched variables in the economy. Their movements are reported almost daily by the news media because they directly affect our everyday lives and have important consequences for the health of the economy. They influence personal decisions such as whether to consume or save, whether to buy a house, and whether to purchase bonds or put funds into a savings account. Interest rates also affect the economic decisions of businesses and households, such as whether to use their funds to invest in new equipment for factories or to save rather than spend their money.

Before we can go on with the study of money, banking, and financial markets, we must understand exactly what the phrase *interest rates* means. In this chapter, we see that a concept known as the *yield to maturity* is the most accurate measure of interest rates; the yield to maturity is what economists mean when they use the term *interest rate*. We'll discuss how the yield to maturity is measured. We'll also see that a bond's interest rate does not necessarily indicate how good an investment the bond is, because what the bond earns (its rate of return) does not necessarily equal its interest rate. Finally, we'll explore the distinction between real interest rates, which are adjusted for inflation, and nominal interest rates, which are not.

Although learning definitions is not always the most exciting of pursuits, it is important to read carefully and understand the concepts presented in this chapter. Not only are they used continually throughout the remainder of this text, but a firm grasp of these terms will give you a clearer understanding of the role that interest rates play in your life as well as in the general economy.

MEASURING INTEREST RATES

Different debt instruments have very different streams of cash payments (known as **cash flows**) to the holder, with very different timing. Thus we first need to understand how we can compare the value of one kind of debt instrument with the value of another before we see how interest rates are measured. To do this, we make use of the concept of *present value*.

Present Value

The concept of **present value** (or **present discounted value**) is based on the commonsense notion that a dollar paid to you one year from now is less valuable than a dollar paid to you today. This notion is true because you can deposit a dollar today in a savings account that earns interest and have more than a dollar in one year. Economists use a more formal definition, as explained in this section.

Let's look at the simplest kind of debt instrument, which we will call a **simple loan**. In this loan, the lender provides the borrower with an amount of funds (called the *principal*) that must be repaid to the lender at the *maturity date*, along with an additional payment for the interest. For example, if you made your friend, Jane, a simple loan of \$100 for one year, you would require her to repay the principal of \$100 in one year's time, along with an additional payment for interest—say, \$10. In the case of a simple loan like this one, the interest payment divided by the amount of the loan is a natural and sensible way to measure the interest rate. This measure of the so-called *simple interest rate*, i, is

$$i = \frac{\$10}{\$100} = 0.10 = 10\%$$

If you make this \$100 loan, at the end of the year you will have \$110, which can be rewritten as

$$\$100 \times (1 + 0.10) = \$110$$

If you then lent out the \$110 at the same interest rate, at the end of the second year you would have

$$\$110 \times (1 + 0.10) = \$121$$

or, equivalently,

$$\$100 \times (1 + 0.10) \times (1 + 0.10) = \$100 \times (1 + 0.10)^2 = \$121$$

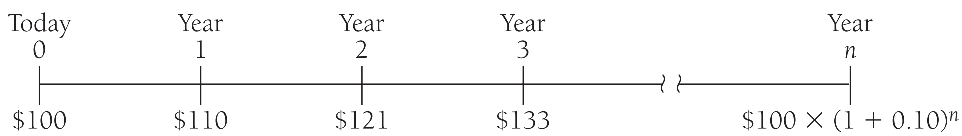
Continuing with the loan again, at the end of the third year you would have

$$\$121 \times (1 + 0.10) = \$100 \times (1 + 0.10)^3 = \$133$$

Generalizing, we can see that at the end of n years, your \$100 would turn into

$$\$100 \times (1 + i)^n$$

The amounts you would have at the end of each year by making the \$100 loan today can be seen in the following timeline:



This timeline clearly indicates that you are just as happy having \$100 today as you will be having \$110 a year from now (of course, as long as you are sure that Jane will pay you back). You are also just as happy having \$100 today as you will be having \$121 two years from now, or \$133 three years from now, or $100 \times (1 + 0.10)^n$ dollars n years from now. The timeline indicates that we can also work backward from future amounts to the present. For example, $\$133 = \$100 \times (1 + 0.10)^3$ three years from now is worth \$100 today, so

$$\$100 = \frac{\$133}{(1 + 0.10)^3}$$

The process of calculating today's value of dollars received in the future, as we have done above, is called *discounting the future*. We can generalize this process by writing today's (present) value of \$100 as PV and the future cash flow (payment) of \$133 as CF, and then replacing 0.10 (the 10% interest rate) by i. This leads to the following formula:

$$PV = \frac{CF}{(1 + i)^n} \tag{1}$$

Intuitively, Equation 1 tells us that if you are promised \$1 of cash flow, for certain, ten years from now, this dollar would not be as valuable to you as \$1 is today because, if you had the \$1 today, you could invest it and end up with more than \$1 in ten years.

The concept of present value is extremely useful, because it allows us to figure out today's value (price) of a credit (debt) market instrument at a given simple interest rate i by just adding up the individual present values of all the future payments received. This information enables us to compare the values of two or more instruments that have very different timing of their payments.

APPLICATION Simple Present Value

What is the present value of \$250 to be paid in two years if the interest rate is 15%?

Solution

The present value would be \$189.04. We find this value by using Equation 1,

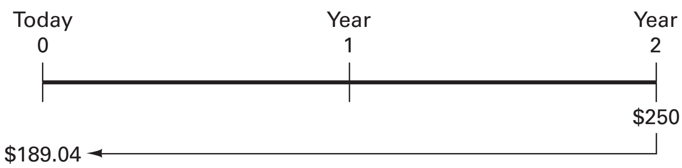
$$PV = \frac{CF}{(1 + i)^n}$$

where

$$\begin{aligned} CF &= \text{cash flow in two years} &&= 250 \\ i &= \text{annual interest rate} &&= 0.15 \\ n &= \text{number of years} &&= 2 \end{aligned}$$

Thus

$$PV = \frac{\$250}{(1 + 0.15)^2} = \frac{\$250}{1.3225} = \$189.04$$



APPLICATION How Much Is That Jackpot Worth?

Assume that you just hit the \$20 million jackpot in the New York State Lottery, which promises you a payment of \$1 million every year for the next twenty years. You are clearly excited, but have you really won \$20 million?

Solution

No, not in the present value sense. In today's dollars, that \$20 million is worth a lot less. If we assume an interest rate of 10% as in the earlier examples, the first payment of \$1 million is clearly worth \$1 million today, but the second payment next year is worth only \$1 million$/(1 + 0.10) =$ \$909,090, a lot less than \$1 million. The following year, the payment is worth \$1 million$/(1 + 0.10)^2 =$ \$826,446 in today's dollars, and so on. When you add up all these amounts, they total \$9.4 million. You are still pretty excited (who wouldn't be?), but because you understand the concept of present value, you recognize that you are the victim of false advertising. In present-value terms, you didn't really win \$20 million, but instead won less than half that amount. ◆

Four Types of Credit Market Instruments

In terms of the timing of their cash flow payments, there are four basic types of credit market instruments.

1. A simple loan, which we have already discussed, in which the lender provides the borrower with an amount of funds that must be repaid to the lender at the maturity date, along with an additional payment for the interest. Many money market instruments are of this type—for example, commercial loans to businesses.
2. A **fixed-payment loan** (also called a **fully amortized loan**) in which the lender provides the borrower with an amount of funds that the borrower must repay by making the same payment, consisting of part of the principal and interest, every period (such as a month) for a set number of years. For example, if you borrow \$1,000, a fixed-payment loan might require you to pay \$126 every year for 25 years. Installment loans (such as auto loans) and mortgages are frequently of the fixed-payment type.
3. A **coupon bond** pays the owner of the bond a fixed interest payment (coupon payment) every year until the maturity date, when a specified final amount (**face value** or **par value**) is repaid. (The coupon payment is so named because in the past, the bondholder obtained payment by clipping a coupon off the bond and sending it to the bond issuer, who then sent the payment to the holder. Today, it is no longer necessary to send in coupons to receive payments.) A coupon bond with \$1,000 face value, for example, might pay you a coupon payment of \$100 per year for ten years, and then repay you the face value amount of \$1,000 at the maturity date. (The face value of a bond is usually in \$1,000 increments.)

   A coupon bond is identified by four pieces of information: First is the bond's face value; second is the corporation or government agency that issues the bond; third is the maturity date of the bond; and fourth is the bond's **coupon rate**, which is the dollar amount of the yearly coupon payment expressed as a percentage of the face value of the bond. In our example, the coupon bond has a yearly coupon payment of \$100 and a face value of \$1,000. The coupon rate is then \$100/\$1,000 = 0.10, or 10%. Capital market instruments such as U.S. Treasury bonds and notes and corporate bonds are examples of coupon bonds.
4. A **discount bond** (also called a **zero-coupon bond**) is bought at a price below its face value (at a discount), and the face value is repaid at the maturity date. Unlike a

coupon bond, a discount bond does not make any interest payments; it just pays off the face value. For example, a one-year discount bond with a face value of \$1,000 might be bought for \$900; in a year's time, the owner would be repaid the face value of \$1,000. U.S. Treasury bills, U.S. savings bonds, and long-term zero-coupon bonds are examples of discount bonds.

These four types of instruments make payments at different times: Simple loans and discount bonds make payments only at their maturity dates, whereas fixed-payment loans and coupon bonds make payments periodically until maturity. How can you decide which of these instruments will provide you with the most income? They all seem so different because they make payments at different times. To solve this problem, we use the concept of present value, explained earlier, to provide us with a procedure for measuring interest rates on these different types of instruments.

Yield to Maturity

Of the several common ways of calculating interest rates, the most important is the **yield to maturity**, which is the interest rate that equates the present value of cash flow payments received from a debt instrument with its value today.[1] Because the concept behind the calculation of the yield to maturity makes good economic sense, economists consider it the most accurate measure of interest rates.

To better understand the yield to maturity, we now look at how it is calculated for the four types of credit market instruments. In all of these examples, the key to understanding the calculation of the yield to maturity is realizing that we are equating today's value of the debt instrument with the present value of all of its future cash flow payments.

Simple Loan Using the concept of present value, the yield to maturity on a simple loan is easy to calculate. For the one-year loan we discussed, today's value is \$100, and the payments in one year's time would be \$110 (the repayment of \$100 plus the interest payment of \$10). We can use this information to solve for the yield to maturity i by recognizing that the present value of the future payments must equal today's value of the loan.

APPLICATION Yield to Maturity on a Simple Loan

If Pete borrows \$100 from his sister and next year she wants \$110 back from him, what is the yield to maturity on this loan?

Solution

The yield to maturity on the loan is 10%.

$$PV = \frac{CF}{(1 + i)^n}$$

where

$$\begin{aligned} PV &= \text{amount borrowed} &&= \$100 \\ CF &= \text{cash flow in one year} &&= \$110 \\ n &= \text{number of years} &&= 1 \end{aligned}$$

[1]In other contexts, the yield to maturity is also called the *internal rate of return*.

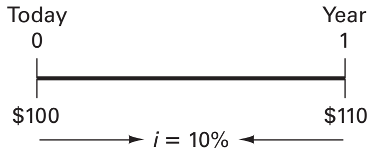
Thus

$$\$100 = \frac{\$110}{(1 + i)}$$

$$(1 + i)\$100 = \$110$$

$$(1 + i) = \frac{\$110}{\$100}$$

$$i = 1.10 - 1 = 0.10 = 10\%$$



This calculation of the yield to maturity should look familiar because it equals the interest payment of \$10 divided by the loan amount of \$100; that is, it equals the simple interest rate on the loan. An important point to recognize is that ***for simple loans, the simple interest rate equals the yield to maturity.*** Hence the same term i is used to denote both the yield to maturity and the simple interest rate.

Fixed-Payment Loan Recall that this type of loan has the same cash flow payment every period throughout the life of the loan. On a fixed-rate mortgage, for example, the borrower makes the same payment to the bank every month until the maturity date, at which time the loan will be completely paid off. To calculate the yield to maturity for a fixed-payment loan, we follow the same strategy that we used for the simple loan—we equate today's value of the loan with its present value. Because the fixed-payment loan involves more than one cash flow payment, the present value of the fixed-payment loan is calculated (using Equation 1) as the sum of the present values of all cash flow payments.

In the case of our earlier example, the loan is \$1,000 and the yearly payment is \$126 for the next 25 years. The present value (*PV*) is calculated as follows: At the end of one year, there is a \$126 payment with a *PV* of $\$126/(1 + i)$; at the end of two years, there is another \$126 payment with a *PV* of $\$126/(1 + i)^2$; and so on until, at the end of the twenty-fifth year, the last payment of \$126 with a *PV* of $\$126/(1 + i)^{25}$ is made. Setting today's value of the loan (\$1,000) equal to the sum of the present values of all the yearly payments gives us

$$\$1{,}000 = \frac{\$126}{1 + i} + \frac{\$126}{(1 + i)^2} + \frac{\$126}{(1 + i)^3} + \dots + \frac{\$126}{(1 + i)^{25}}$$

More generally, for any fixed-payment loan,

$$LV = \frac{FP}{(1 + i)} + \frac{FP}{(1 + i)^2} + \frac{FP}{(1 + i)^3} + \dots + \frac{FP}{(1 + i)^n} \qquad (2)$$

where LV = loan value
FP = fixed yearly payment
n = number of years until maturity

For a fixed-payment loan, the loan value, the fixed yearly payment, and the number of years until maturity are known quantities, and only the yield to maturity is not. So we can solve this equation for the yield to maturity i. Because this calculation is not easy, many financial calculators have programs that enable you to find i given the loan's numbers for LV, FP, and n. For example, in the case of a 25-year \$1,000 loan with yearly payments of \$85.81, the yield to maturity that results from solving Equation 2 is 7%. Real estate brokers always have a financial calculator that can solve such equations handy, so that they can immediately tell the prospective house buyer exactly what the yearly (or monthly) payments will be if the house purchase is financed by a mortgage.

APPLICATION Yield to Maturity and the Yearly Payment on a Fixed-Payment Loan

You decide to purchase a new home and need a \$100,000 mortgage. You take out a loan from the bank that has an interest rate of 7%. What is the yearly payment to the bank if you wish to pay off the loan in twenty years?

Solution

The yearly payment to the bank is \$9,439.29.

$$LV = \frac{FP}{(1+i)} + \frac{FP}{(1+i)^2} + \frac{FP}{(1+i)^3} + \dots + \frac{FP}{(1+i)^n}$$

where LV = loan value amount = 100,000
i = annual interest rate = 0.07
n = number of years = 20

Thus

$$\$100{,}000 = \frac{FP}{(1+0.07)} + \frac{FP}{(1+0.07)^2} + \frac{FP}{(1+0.07)^3} + \dots + \frac{FP}{(1+0.07)^{20}}$$

To find the monthly payment for the loan using a financial calculator:

n = number of years = 20
PV = amount of the loan (LV) = 100,000
FV = amount of the loan after 20 years = 0
i = annual interest rate = 0.07

Then push the *PMT* button to get the fixed yearly payment (FP) = \$9,439.29. ◆

Coupon Bond To calculate the yield to maturity for a coupon bond, follow the same strategy used for the fixed-payment loan: Equate today's value of the bond with its present value. Because coupon bonds also have more than one cash flow payment, the present value of the bond is calculated as the sum of the present values of all the coupon payments plus the present value of the final payment of the face value of the bond.

The present value of a \$1,000-face-value bond with ten years to maturity and yearly coupon payments of \$100 (a 10% coupon rate) can be calculated as follows: At the end of one year, there is a \$100 coupon payment with a *PV* of $\$100/(1 + i)$; at the end of the second year, there is another \$100 coupon payment with a *PV* of $\$100/(1 + i)^2$; and so on until, at maturity, there is a \$100 coupon payment with a *PV* of $\$100/(1 + i)^{10}$ plus the repayment of the \$1,000 face value with a *PV* of $\$1{,}000/(1 + i)^{10}$. Setting today's value of the bond (its current price, denoted by *P*) equal to the sum of the present values of all the cash flow payments for the bond gives

$$P = \frac{\$100}{1 + i} + \frac{\$100}{(1 + i)^2} + \frac{\$100}{(1 + i)^3} + \cdots + \frac{\$100}{(1 + i)^{10}} + \frac{\$1{,}000}{(1 + i)^{10}}$$

More generally, for any coupon bond,[2]

$$P = \frac{C}{1 + i} + \frac{C}{(1 + i)^2} + \frac{C}{(1 + i)^3} + \cdots + \frac{C}{(1 + i)^n} + \frac{F}{(1 + i)^n} \tag{3}$$

where
P = price of the coupon bond
C = yearly coupon payment
F = face value of the bond
n = years to maturity date

In Equation 3, the coupon payment, the face value, the years to maturity, and the price of the bond are known quantities, and only the yield to maturity is not. Hence we can solve this equation for the yield to maturity *i*. As in the case of the fixed-payment loan, this calculation is not easy, so business-oriented software and financial calculators have built-in programs that solve the equation for you.

APPLICATION Yield to Maturity and Bond Price for a Coupon Bond

Find the price of a 10% coupon bond with a face value of \$1000, a 12.25% yield to maturity, and eight years to maturity.

Solution

The price of the bond is \$889.20. To solve using a financial calculator:

$$\begin{aligned} n &= \text{years to maturity} &&= 8 \\ FV &= \text{face value of the bond } (F) &&= 1{,}000 \\ i &= \text{annual interest rate} &&= 12.25\% \\ PMT &= \text{yearly coupon payments} (C) &&= 100 \end{aligned}$$

Then push the *PV* button to get the price of the bond = \$889.20.

[2]Most coupon bonds actually make coupon payments on a semiannual basis rather than once a year as we assumed here. The effect on the calculations is very slight and will be ignored here.

Alternatively, you could solve for the yield to maturity given the bond price by entering $889.20 for *PV* and pushing the *i* button to get a yield to maturity of 12.25%. ◆

Table 1 shows the yields to maturity calculated for several bond prices. Three interesting facts emerge:

1. When the coupon bond is priced at its face value, the yield to maturity equals the coupon rate.
2. The price of a coupon bond and the yield to maturity are negatively related; that is, as the yield to maturity rises, the price of the bond falls. As the yield to maturity falls, the price of the bond rises.
3. The yield to maturity is greater than the coupon rate when the bond price is below its face value and is less than the coupon rate when the bond price is above its face value.

These three facts are true for any coupon bond and are really not surprising if you think about the reasoning behind the calculation of the yield to maturity. When you put $1,000 in a bank account with an interest rate of 10%, you can take out $100 every year and you will be left with the $1,000 at the end of ten years. This process is similar to buying the $1,000 bond with a 10% coupon rate analyzed in Table 1, which pays a $100 coupon payment every year and then repays $1,000 at the end of ten years. If the bond is purchased at the par value of $1,000, its yield to maturity must equal 10%, which is also equal to the coupon rate of 10%. The same reasoning applied to any coupon bond demonstrates that if the coupon bond is purchased at its par value, the yield to maturity and the coupon rate must be equal.

It is a straightforward process to show that the bond price and the yield to maturity are negatively correlated. As *i*, the yield to maturity, increases, all denominators in the bond price formula (Equation 3) must necessarily increase, because the rise in *i* lowers the present value of all future cash flow payments for this bond. Hence a rise in the interest rate, as measured by the yield to maturity, means that the price of the bond must fall. Another way to explain why the bond price falls when the interest rate rises is to consider that a higher interest rate implies that the future coupon payments and final payment are worth less when discounted back to the present; hence the price of the bond must be lower.

The third fact, that the yield to maturity is greater than the coupon rate when the bond price is below its par value, follows directly from facts 1 and 2. When the yield to maturity equals the coupon rate, then the bond price is at the face value; when the

TABLE 1 Yields to Maturity on a 10%-Coupon-Rate Bond Maturing in Ten Years (Face Value = $1,000)

Price of Bond ($)	Yield to Maturity (%)
1,200	7.13
1,100	8.48
1,000	10.00
900	11.75
800	13.81

yield to maturity rises above the coupon rate, the bond price necessarily falls and so must be below the face value of the bond.

One special case of a coupon bond is worth discussing here because its yield to maturity is particularly easy to calculate. This bond is called a **consol** or a **perpetuity**; it is a perpetual bond with no maturity date and no repayment of principal that makes fixed coupon payments of $\$C$ forever. Consols were first sold by the British Treasury during the Napoleonic Wars and are still traded today; they are quite rare, however, in American capital markets. The formula in Equation 3 for the price of the consol Pc simplifies to the following:[3]

$$P_c = \frac{C}{i_c} \tag{4}$$

where P_c = price of the perpetuity (consol)
C = yearly payment
i_c = yield to maturity of the perpetuity (consol)

One nice feature of perpetuities is that you can immediately see that as i_c increases, the price of the bond falls. For example, if a perpetuity pays \$100 per year forever and the interest rate is 10%, its price will be $\$1{,}000 = \$100/0.10$. If the interest rate rises to 20%, its price will fall to $\$500 = \$100/0.20$. We can rewrite this formula as

$$i_c = \frac{C}{P_c} \tag{5}$$

APPLICATION Yield to Maturity on a Perpetuity

What is the yield to maturity on a bond that has a price of \$2,000 and pays \$100 of interest annually, forever?

Solution

The yield to maturity is 5%.

$$i_c = \frac{C}{P_c}$$

[3]The bond price formula for a consol is

$$P = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \ \ldots$$

which can be written as

$$P = C(x + x^2 + x^3 + \ \ldots)$$

in which $x = 1/(1 + i)$. The formula for an infinite sum is

$$1 + x + x^2 + x^3 + \ \ldots = \frac{1}{1-x} \text{ for } x < 1$$

and so

$$P = C\left(\frac{1}{1-x} - 1\right) = C\left[\frac{1}{1 - 1/(1+i)} - 1\right]$$

which by suitable algebraic manipulation becomes

$$P = C\left(\frac{1+i}{i} - \frac{i}{i}\right) = \frac{C}{i}$$

where C = yearly payment = \$100

P_c = price of perpetuity (consol) = \$2,000

Thus

$$i_c = \frac{\$100}{\$2,000}$$

$$i_c = 0.05 = 5\%$$ ◆

The formula given in Equation 5, which describes the calculation of the yield to maturity for a perpetuity, also provides a useful approximation for the yield to maturity on coupon bonds. When a coupon bond has a long term to maturity (say, twenty years or more), it is very much like a perpetuity, which pays coupon payments forever. This is because the cash flows more than twenty years in the future have such small present discounted values that the value of a long-term coupon bond is very close to the value of a perpetuity with the same coupon rate. Thus i_c in Equation 5 will be very close to the yield to maturity for any long-term bond. For this reason, i_c, the yearly coupon payment divided by the price of the security, has been given the name **current yield** and is frequently used as an approximation to describe interest rates on long-term bonds.

Discount Bond The yield-to-maturity calculation for a discount bond is similar to that for a simple loan. Let's consider a discount bond such as a one-year U.S. Treasury bill that pays a face value of \$1,000 in one year's time but today has a price of \$900.

APPLICATION Yield to Maturity on a Discount Bond

What is the yield to maturity on a one-year, \$1,000 Treasury bill with a current price of \$900?

Solution

The yield to maturity is 11.1%.

Using the present value formula,

$$PV = \frac{CF}{(1 + i)^n}$$

and recognizing that the present value (PV) is the current price of \$900, the cash flow in one year is \$1,000, and the number of years is 1, we can write:

$$\$900 = \frac{\$1,000}{1 + i}$$

Solving for i, we get

$$(1 + i) \times \$900 = \$1,000$$
$$\$900 + \$900i = \$1,000$$
$$\$900i = \$1,000 - \$900$$
$$i = \frac{\$1,000 - \$900}{\$900} = 0.111 = 11.1\%$$ ◆

As we just saw in the preceding application, the yield to maturity for a one-year discount bond equals the increase in price over the year, \$1,000 – \$900, divided by the initial price, \$900. Hence, more generally, for any one-year discount bond, the yield to maturity can be written as

$$i = \frac{F - P}{P} \tag{6}$$

where F = face value of the discount bond
P = current price of the discount bond

An important feature of this equation is that it indicates that, for a discount bond, the yield to maturity is negatively related to the current bond price. This is the same conclusion that we reached for a coupon bond. Equation 6 shows that a rise in the bond price—say, from \$900 to \$950—means that the bond will have a smaller increase in its price at maturity and so the yield to maturity will fall, from 11.1% to 5.3% in our example. Similarly, a fall in the yield to maturity means that the current price of the discount bond has risen.

Summary The concept of present value tells you that a dollar in the future is not as valuable to you as a dollar today because you can earn interest on a dollar you have today. Specifically, a dollar received n years from now is worth only $\$1/(1 + i)^n$ today. The present value of a set of future cash flow payments on a debt instrument equals the sum of the present values of each of the future payments. The yield to maturity for an instrument is the interest rate that equates the present value of the future payments on that instrument to its value today. Because the procedure for calculating the yield to maturity is based on sound economic principles, the yield to maturity is the measure that economists think most accurately describes the interest rate.

Our calculations of the yield to maturity for a variety of bonds reveal the important fact that ***current bond prices and interest rates are negatively related: When the interest rate rises, the price of the bond falls, and vice versa***.

THE DISTINCTION BETWEEN INTEREST RATES AND RETURNS

Many people think that the interest rate on a bond tells them all they need to know about how well off they are as a result of owning it. If Irving the Investor thinks he is well off when he owns a long-term bond yielding a 10% interest rate, and the interest rate then rises to 20%, he will have a rude awakening: As we will shortly see, if he has to sell the bond, Irving will lose his shirt! How well a person does financially by holding a bond or any other security over a particular time period is accurately measured by the security's **return**, or, in more precise terminology, the **rate of return**. We will use the concept of *return* continually throughout this book: Understanding this concept will make the material presented later in the book easier to follow.

For any security, the rate of return is defined as the amount of each payment to the owner plus the change in the security's value, expressed as a fraction of its purchase price. To make this definition clearer, let us see what the return would look like for a \$1,000-face-value coupon bond with a coupon rate of 10% that is bought for \$1,000, held for one year, and then sold for \$1,200. The payments to the owner are the yearly coupon payments of \$100, and the change in the bond's value is \$1,200 − \$1,000 = \$200. Adding these values together and expressing them as a fraction of the purchase price of \$1,000 gives us the one-year holding-period return for this bond:

$$\frac{\$100 + \$200}{\$1{,}000} = \frac{\$300}{\$1{,}000} = 0.30 = 30\%$$

You may have noticed something quite surprising about the return that we just calculated: It equals 30%, yet as Table 1 indicates, initially the yield to maturity was only 10%. This discrepancy demonstrates that ***the return on a bond will not necessarily equal the yield to maturity on that bond***. We now see that the distinction between interest rate and return can be important, although for many securities the two may be closely related.

More generally, the return on a bond held from time t to time $t + 1$ can be written as

$$R = \frac{C + P_{t+1} - P_t}{P_t} \tag{7}$$

where
R = return from holding the bond from time t to time $t + 1$
P_t = price of the bond at time t
P_{t+1} = price of the bond at time $t + 1$
C = coupon payment

A convenient way to rewrite the return formula in Equation 7 is to recognize that it can be split into two separate terms:

$$R = \frac{C}{P_t} + \frac{P_{t+1} - P_t}{P_t}$$

The first term is the current yield i_c (the coupon payment over the purchase price):

$$\frac{C}{P_t} = i_c$$

The second term is the **rate of capital gain**, or the change in the bond's price relative to the initial purchase price:

$$\frac{P_{t+1} - P_t}{P_t} = g$$

where g is the rate of capital gain. Equation 7 can then be rewritten as

$$R = i_c + g \tag{8}$$

which shows that the return on a bond is the current yield i_c plus the rate of capital gain g. This rewritten formula illustrates the point we just discovered: Even for a bond for which the current yield i_c is an accurate measure of the yield to maturity, the return can differ substantially from the interest rate. Returns will differ substantially from the interest rate if the price of the bond experiences sizable fluctuations that produce substantial capital gains or losses.

TABLE 2 One-Year Returns on Different-Maturity 10%-Coupon-Rate Bonds When Interest Rates Rise from 10% to 20%

(1) Years to Maturity When Bond Is Purchased	(2) Initial Current Yield (%)	(3) Initial Price ($)	(4) Price Next Year* ($)	(5) Rate of Capital Gain (%)	(6) Rate of Return [col (2) + col (5)] (%)
30	10	1,000	503	−49.7	−39.7
20	10	1,000	516	−48.4	−38.4
10	10	1,000	597	−40.3	−30.3
5	10	1,000	741	−25.9	−15.9
2	10	1,000	917	−8.3	+1.7
1	10	1,000	1,000	0.0	+10.0

*Calculated with a financial calculator, using Equation 3.

To explore this point even further, let's look at what happens to the returns on bonds of different maturities when interest rates rise. Table 2 calculates the one-year returns, using Equation 8 above, on several 10%-coupon-rate bonds, all purchased at par, when interest rates on all these bonds rise from 10% to 20%. Several key findings from this table are generally true of all bonds:

- The only bonds whose returns will equal their initial yields to maturity are those whose times to maturity are the same as their holding periods (see, for example, the last bond in Table 2).
- A rise in interest rates is associated with a fall in bond prices, resulting in capital losses on bonds whose terms to maturity are longer than their holding periods.
- The more distant a bond's maturity date, the greater the size of the percentage price change associated with an interest rate change.
- The more distant a bond's maturity date, the lower the rate of return that occurs as a result of an increase in the interest rate.
- Even though a bond may have a substantial initial interest rate, its return can turn out to be negative if interest rates rise.

At first, students are frequently puzzled (as was poor Irving the Investor) that a rise in interest rates can mean that a bond has been a poor investment. The trick to understanding this fact is to recognize Irving has already purchased the bond, so that a rise in the interest rate means that the price of the bond Irving is holding falls and he experiences a capital loss. If this loss is large enough, the bond can be a poor investment indeed. For example, we see in Table 2 that the bond that has 30 years to maturity when purchased has a capital loss of 49.7% when the interest rate rises from 10% to 20%. This loss is so large that it exceeds the current yield of 10%, resulting in a negative return (loss) of –39.7%. If Irving does not sell the bond, his capital loss is often referred to as a "paper loss." This is a loss nonetheless because if Irving had not bought this bond and had instead put his money in the bank, he would now be able to buy more bonds at their lower price than he presently owns.

Maturity and the Volatility of Bond Returns: Interest-Rate Risk

The finding that the prices of longer-maturity bonds respond more dramatically to changes in interest rates helps explain an important fact about the behavior of bond markets: ***Prices and returns for long-term bonds are more volatile than those for shorter-term bonds***. Price changes of +20% and −20% within a year, with corresponding variations in returns, are common for bonds that are more than twenty years away from maturity.

We now see that changes in interest rates make investments in long-term bonds quite risky. Indeed, the risk level associated with an asset's return that results from interest-rate changes is so important that it has been given a special name, **interest-rate risk**.[4] Dealing with interest-rate risk is a major concern of managers of financial institutions and investors, as we will see in later chapters.

Although long-term debt instruments have substantial interest-rate risk, short-term debt instruments do not. Indeed, bonds with a maturity term that is as short as the holding period have no interest-rate risk.[5] We see this for the coupon bond at the bottom of Table 2, which has no uncertainty about the rate of return because it equals the yield to maturity, which is known at the time the bond is purchased. The key to understanding why there is no interest-rate risk for *any* bond whose time to maturity matches the holding period is to recognize that (in this case) the price at the end of the holding period is already fixed at the face value. A change in interest rates can then have no effect on the price at the end of the holding period for these bonds, and the return will therefore be equal to the yield to maturity, which is known at the time the bond is purchased.[6]

[4]Interest-rate risk can be quantitatively measured using the concept of *duration*. This concept and its calculation are discussed in an appendix to this chapter, which can be found on the Companion Website at http://www.pearsonglobaleditions.com/Mishkin.

[5]The statement that there is no interest-rate risk for any bond whose time to maturity matches the holding period is literally true only for discount (zero-coupon) bonds that make no intermediate cash payments before the holding period is over. A coupon bond that makes an intermediate cash payment before the holding period is over requires that this payment be reinvested. Because the interest rate at which this payment can be reinvested is uncertain, some uncertainty exists about the return on this coupon bond even when the time to maturity equals the holding period. However, the riskiness of the return on a coupon bond from reinvesting the coupon payments is typically quite small, so a coupon bond with a time to maturity equal to its holding period still has very little risk.

[6]In this text we assume that holding periods on all short-term bonds are equal to the term to maturity, and thus the bonds are not subject to interest-rate risk. However, if the term to maturity of the bond is shorter than an investor's holding period, the investor is exposed to a type of interest-rate risk called *reinvestment risk*. Reinvestment risk occurs because the proceeds from the short-term bond need to be reinvested at a future interest rate that is uncertain.

To understand reinvestment risk, suppose that Irving the Investor has a holding period of two years and decides to purchase a $1,000, one-year bond at face value and then another one at the end of the first year. If the initial interest rate is 10%, Irving will have $1,100 at the end of the first year. If the interest rate rises to 20%, as in Table 2, Irving will find that buying $1,100 worth of another one-year bond will leave him at the end of the second year with $\$1{,}100 \times (1 + 0.20) = \$1{,}320$. Thus Irving's two-year return will be $(\$1{,}320 - \$,1000)/1{,}000 = 0.32 =$ 32%, which equals 14.9% at an annual rate. In this case, Irving has earned more by buying the one-year bonds than he would have if he had initially purchased a two-year bond with an interest rate of 10%. Thus, when Irving has a holding period that is longer than the term to maturity of the bonds he purchases, he benefits from a rise in interest rates. Conversely, if interest rates fall to 5%, Irving will have only $1,155 at the end of two years: $\$1{,}100 \times (1 + 0.05)$. His two-year return will be $(\$1{,}555 - \$1{,}000)/1{,}000 = 0.155 = 15.5\%$, which is 7.2% at an annual rate. With a holding period greater than the term to maturity of the bond, Irving now loses from a decline in interest rates.

We have seen that when the holding period is longer than the term to maturity of a bond, the return is uncertain because the future interest rate when reinvestment occurs is also uncertain—in short, there is reinvestment risk. We also see that if the holding period is longer than the term to maturity of the bond, the investor benefits from a rise in interest rates and is hurt by a fall in interest rates.

Summary

The return on a bond, which tells you how good an investment it has been over the holding period, is equal to the yield to maturity in only one special case—when the holding period and the term to maturity of the bond are identical. Bonds whose terms to maturity are longer than their holding periods are subject to interest-rate risk: Changes in interest rates lead to capital gains and losses that produce substantial differences between the return and the yield to maturity known at the time the bond is purchased. Interest-rate risk is especially important for long-term bonds, on which capital gains and losses can be substantial. This is why long-term bonds are not considered safe assets with a sure return over short holding periods.

THE DISTINCTION BETWEEN REAL AND NOMINAL INTEREST RATES

So far in our discussion of interest rates, we have ignored the effects of inflation on the cost of borrowing. What we up to this point have been calling the interest rate makes no allowance for inflation, and it is more precisely referred to as the **nominal interest rate**. We must distinguish the nominal interest rate from the **real interest rate**, which is the interest rate that is adjusted by subtracting expected changes in the price level (inflation) so that it more accurately reflects the true cost of borrowing. This interest rate is more precisely referred to as the *ex ante real interest rate* because it is adjusted for *expected* changes in the price level. The ex ante real interest rate is veryimportant to economic decisions, and typically it is what economists mean when they make reference to the "real" interest rate. The interest rate that is adjusted for *actual* changes in the price level is called the *ex post real interest rate.* It describes how well a lender has done in real terms *after the fact.*

The real interest rate is more accurately defined from the *Fisher equation*, named for Irving Fisher, one of the great monetary economists of the twentieth century. The Fisher equation states that the nominal interest rate i equals the real interest rate r plus the expected rate of inflation π^e:[7]

$$i = r + \pi^e \tag{9}$$

Rearranging terms, we find that the real interest rate equals the nominal interest rate minus the expected inflation rate:

$$r = i - \pi^e \tag{10}$$

To see why this definition makes sense, let's first consider a situation in which you have made a simple one-year loan with a 5% interest rate $(i = 5\%)$, and you expect the price level to rise by 3% over the course of the year $(\pi^e = 3\%)$. As a result of making

[7]A more precise formulation of the Fisher equation is

$$i = r + \pi^e + (r \times \pi^e)$$

because

$$1 + i = (1 + r)(1 + \pi^e) = 1 + r + \pi^e + (r \times \pi^e)$$

and subtracting 1 from both sides gives us the first equation. For small values of r and π^e, the term $r \times \pi^e$ is so small that we ignore it, as in the text.

the loan, at the end of the year you expect to have 2% more in **real terms**—that is, in terms of real goods and services you can buy. In this case, the interest rate you expect to earn in terms of real goods and services is 2%:

$$r = 5\% - 3\% = 2\%$$

as indicated by Equation 10.[8]

APPLICATION Calculating Real Interest Rates

What is the real interest rate if the nominal interest rate is 8% and the expected inflation rate is 10% over the course of a year?

Solution

The real interest rate is −2%. Although you will be receiving 8% more dollars at the end of the year, you will be paying 10% more for goods. The result is that you will be able to buy 2% fewer goods at the end of the year, and you will be 2% worse off in real terms. Mathematically,

$$r = i - \pi^e$$

where
$$i = \text{nominal interest rate} = 0.08$$
$$\pi^e = \text{expected inflation rate} = 0.10$$

Thus

$$r = 0.08 - 0.10 = -0.02 = -2\%$$

◆

As a lender, you are clearly less eager to make a loan in this case, because in terms of real goods and services you have actually earned a negative interest rate of 2%. By contrast, as a borrower, you fare quite well because at the end of the year, the amounts you will have to pay back will be worth 2% less in terms of goods and services—you

[8]Because most interest income in the United States is subject to federal income taxes, the true earnings in real terms from holding a debt instrument are not reflected by the real interest rate defined by the Fisher equation but rather by the *after-tax real interest rate,* which equals the nominal interest rate *after income tax payments have been subtracted,* minus the expected inflation rate. For a person facing a 30% tax rate, the after-tax interest rate earned on a bond yielding 10% is only 7% because 30% of the interest income must be paid to the Internal Revenue Service. Thus the after-tax real interest rate on this bond when expected inflation is 5% equals $2\% (=7\% - 5\%)$. More generally, the after-tax real interest rate can be expressed as

$$i(1 - \tau) - \pi^e$$

where τ = the income tax rate.

This formula for the after-tax real interest rate also provides a better measure of the effective cost of borrowing for many corporations and homeowners in the United States because, in calculating income taxes, they can deduct interest payments on loans from their income. Thus, if you face a 30% tax rate and take out a mortgage loan with a 10% interest rate, you are able to deduct the 10% interest payment and lower your taxes by 30% of this amount. Your after-tax nominal cost of borrowing is then 7% (10% minus 30% of the 10% interest payment), and when the expected inflation rate is 5%, the effective cost of borrowing in real terms is again $2\% (=7\% - 5\%)$.

As the example (and the formula) indicates, after-tax real interest rates are always below the real interest rate defined by the Fisher equation. For a further discussion of measures of after-tax real interest rates, see Frederic S. Mishkin, "The Real Interest Rate: An Empirical Investigation," *Carnegie-Rochester Conference Series on Public Policy* 15 (1981): 151–200.

MyEconLab Mini-lecture MyEconLab Real-time data

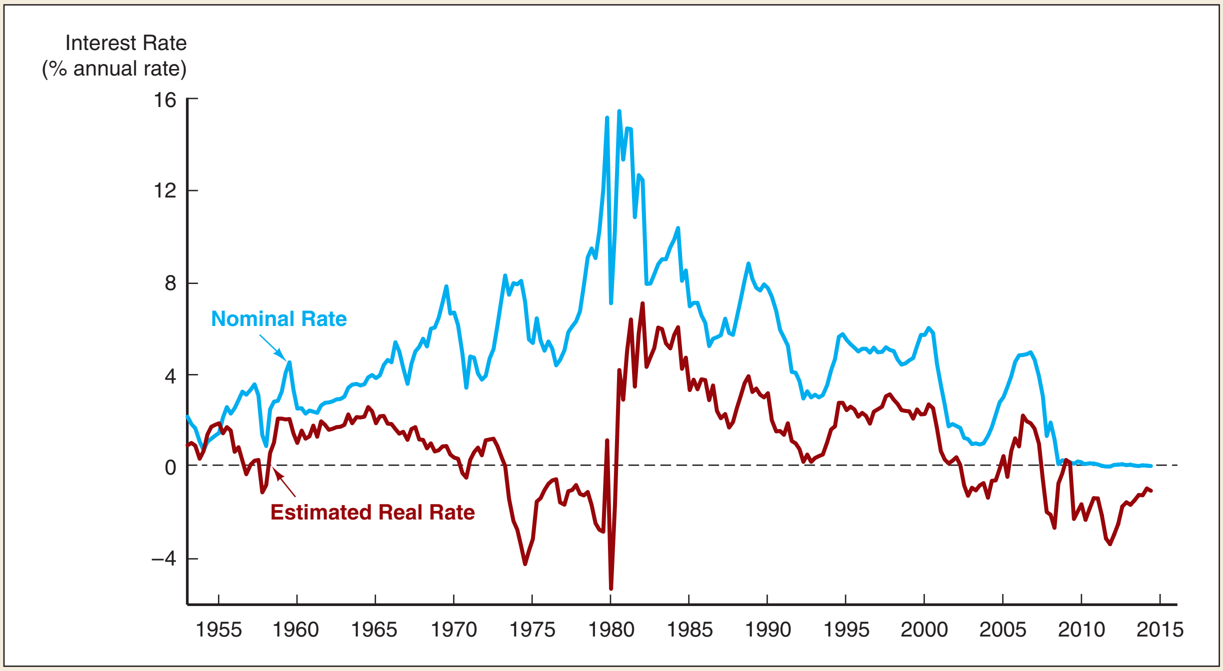


FIGURE 1 Real and Nominal Interest Rates (Three-Month Treasury Bill), 1953–2014
Nominal and real interest rates often do not move together. When U.S. nominal rates were high in the 1970s, real rates were actually extremely low—often negative.
Sources: Nominal rates from Federal Reserve Bank of St. Louis FRED database: http://research.stlouisfed.org/fred2/. The real rate is constructed using the procedure outlined in Frederic S. Mishkin, "The Real Interest Rate: An Empirical Investigation," *Carnegie-Rochester Conference Series on Public Policy* 15 (1981): 151–200. This procedure involves estimating expected inflation as a function of past interest rates, inflation, and time trends, and then subtracting the expected inflation measure from the nominal interest rate.

as the borrower will be ahead by 2% in real terms. ***When the real interest rate is low, there are greater incentives to borrow and fewer incentives to lend.***

A similar distinction can be made between nominal returns and real returns. Nominal returns, which do not allow for inflation, are what we have been referring to as simply "returns." When inflation is subtracted from a nominal return, we have the real return, which indicates the amount of extra goods and services that we can purchase as a result of holding the security.

The distinction between real and nominal interest rates is important because the real interest rate, which reflects the real cost of borrowing, is likely to be a better indicator of the incentives to borrow and lend. It appears to be a better guide to how people will respond to what is happening in credit markets. Figure 1, which presents estimates from 1953 to 2014 of the real and nominal interest rates on three-month U.S. Treasury bills, shows us that nominal and real rates usually move together but do not always do so. (This is also true for nominal and real interest rates in the rest of the world.) In particular, when nominal rates in the United States were high in the 1970s, real rates were actually extremely low—often negative. By the standard of nominal interest rates, you would have thought that credit market conditions were tight during this period because it was expensive to borrow. However, the estimates of the real rates indicate that you would have been mistaken. In real terms, the cost of borrowing was actually quite low.

SUMMARY

1. The yield to maturity, which is the measure that most accurately reflects the interest rate, is the interest rate that equates the present value of future payments of a debt instrument with the instrument's value today. Application of this principle reveals that bond prices and interest rates are negatively correlated: When the interest rate rises, the price of the bond must fall, and vice versa.
2. The return on a security, which tells you how well you have done by holding the security over a stated period of time, can differ substantially from the interest rate as measured by the yield to maturity. Long-term bond prices experience substantial fluctuations when interest rates change and thus bear interest-rate risk. The resulting capital gains and losses can be large, which is why long-term bonds are not considered safe assets with a sure return.
3. The real interest rate is defined as the nominal interest rate minus the expected rate of inflation. It is both a better measure of the incentives to borrow and lend and a more accurate indicator of the tightness of credit market conditions than is the nominal interest rate.

KEY TERMS

cash flows, p. 110
consol or perpetuity, p. 119
coupon bond, p. 113
coupon rate, p. 113
current yield, p. 120
discount bond (zero-coupon bond), p. 113
face value (par value), p. 113
fixed-payment loan (fully amortized loan), p. 113
interest-rate risk, p. 124
nominal interest rate, p. 125
present value (present discounted value), p. 110
rate of capital gain, p. 122
real interest rate, p. 125
real terms, p. 126
return (rate of return), p. 121
simple loan, p. 111
yield to maturity, p. 114

QUESTIONS

Select questions are available in MyEconLab at http://www.myeconlab.com.

1. Would $175, to be received in exactly one year, be worth more to you today when the interest rate is 15% or when it is 20%?
2. Write down the formula that is used to calculate the yield to maturity on a twenty-year 12% coupon bond with a $1,000 face value that sells for $2,500.
3. To help pay for college, you have just taken out a $1,000 government loan that makes you pay $126 per year for 25 years. However, you don't have to start making these payments until you graduate from college two years from now. Why is the yield to maturity necessarily less than 12%? (This is the yield to maturity on a normal $1,000 fixed-payment loan on which you pay $126 per year for 25 years.)
4. Do bondholders fare better when the yield to maturity increases or when it decreases? Why?
5. A financial adviser has just given you the following advice: "Long-term bonds are a great investment because their interest rate is over 20%." Is the financial adviser necessarily right?
6. If mortgage rates rise from 5% to 10% but the expected rate of increase in housing prices rises from 2% to 9%, are people more or less likely to buy houses?
7. When is the current yield a good approximation of the yield to maturity?
8. Why would a government choose to issue a perpetuity, which requires payments forever, instead of a terminal loan, such as a fixed-payment loan, discount bond, or coupon bond?
9. Under what conditions will a discount bond have a negative nominal interest rate? Is it possible for a coupon bond or a perpetuity to have a negative nominal interest rate?
10. True or False: With a discount bond, the return on the bond is equal to the rate of capital gain.

11. If interest rates decline, which would you rather be holding, long-term bonds or short-term bonds? Why? Which type of bond has the greater interest-rate risk?

12. Interest rates were lower in the mid-1980s than in the late 1970s, yet many economists have commented that real interest rates were actually much higher in the mid-1980s than in the late 1970s. Does this make sense? Do you think that these economists are right?

13. Retired persons often have much of their wealth placed in savings accounts and other interest-bearing investments, and complain whenever interest rates are low. Do they have a valid complaint?

APPLIED PROBLEMS

Select applied problems are available in MyEconLab *at* http://www.myeconlab.com.

14. If the interest rate is 15%, what is the present value of a security that pays you \$1,100 next year, \$1,250 the year after, and \$1,347 the year after that?

15. Calculate the present value of a \$1,300 discount bond with seven years to maturity if the yield to maturity is 8%.

16. A lottery claims its grand prize is \$15 million, payable over 5 years at \$3,000,000 per year. If the first payment is made immediately, what is this grand prize really worth? Use an interest rate of 7%.

17. What is the yield to maturity on a \$10,000-face-value discount bond maturing in one year that sells for \$9,523.81?

18. What is the yield to maturity (YTM) on a simple loan for \$1,500 that requires a repayment of \$15,000 in five years' time?

19. Which \$10,000 bond has the higher yield to maturity, a twenty-year bond selling for \$8,000 with a current yield of 20% or a one-year bond selling for \$8,000 with a current yield of 10%?

20. Consider a bond with a 6% annual coupon and a face value of \$1,000. Complete the following table. What relationships do you observe between years to maturity, yield to maturity, and the current price?

Years to Maturity	Yield to Maturity	Current Price
2	4%	
2	6%	
3	6%	
5	4%	
5	8%	

21. Consider a coupon bond that has a \$900 par value and a coupon rate of 6%. The bond is currently selling for \$860.15 and has two years to maturity. What is the bond's yield to maturity (YTM)?

22. What is the price of a perpetuity that has a coupon of \$70 per year and a yield to maturity of 1.5%? If the yield to maturity doubles, what will happen to the perpetuity's price?

23. Property taxes in a particular district are 2% of the purchase price of a home every year. If you just purchased a \$150,000 home, what is the present value of all the future property tax payments? Assume that the house remains worth \$150,000 forever, property tax rates never change, and a 4% interest rate is used for discounting.

24. A \$1,100-face-value bond has a 5% coupon rate, its current price is \$1,040, and it is expected to increase to \$1070 next year. Calculate the current yield, the expected rate of capital gains, and the expected rate of return.

25. Assume you just deposited \$1,250 into a bank account. The current real interest rate is 1%, and inflation is expected to be 5% over the next year. What nominal rate would you require from the bank over the next year? How much money will you have at the end of one year? If you are saving to buy a fancy bicycle that currently sells for \$1,300, will you have enough money to buy it?

DATA ANALYSIS PROBLEMS

The Problems update with real-time data in MyEconLab and are available for practice or instructor assignment.

1. Go to the St. Louis Federal Reserve FRED database, and find data on the interest rate on a four-year auto loan (TERMCBAUTO48NS). Assume that you borrow $20,000 to purchase a new automobile and that you finance it with a four-year loan at the most recent interest rate given in the database. If you make one payment per year for four years, what will the yearly payment be? What is the total amount that will be paid out on the $20,000 loan?
2. The U.S. Treasury issues some bonds as *Treasury Inflation Indexed Securities, or TIIS*, which are bonds adjusted for inflation; hence the yields can be roughly interpreted as real interest rates. Go to the St. Louis Federal Reserve FRED database, and find data on the following TIIS bonds and their nominal counterparts. Then answer the questions below.
   - 5 year U.S. treasury (DGS5) and 5 year TIIS (DFII5)
   - 7 year U.S. treasury (DGS7) and 7 year TIIS (DFII7)
   - 10 year U.S. treasury (DGS10) and 10 year TIIS (DFII10)
   - 20 year U.S. treasury (DGS20) and 20 year TIIS (DFII20)
   - 30 year U.S. treasury (DGS30) and 30 year TIIS (DFII30)

   a. Following the Great Recession of 2008–2009, the 5, 7, 10, and even the 20 year TIIS yields became negative for a period of time. How is this possible?

   b. Using the most recent data available, calculate the difference between the yields for each of the pairs of bonds (DGS5 – DFII5, etc.) listed above. What does this difference represent?

   c. Based on your answer to part (b), are there significant variations among the differences in the bond-pair yields? Interpret the magnitude of the variation in differences among the pairs.

WEB EXERCISES

1. In this chapter, we discussed long-term bonds as if there were only one type, coupon bonds. In fact, investors can also purchase long-term discount bonds. A discount bond is sold at a low price, and the whole return comes in the form of a price appreciation. You can easily compute the current price of a discount bond by using the financial calculator at http://www.treasurydirect.gov/indiv/tools/tools_savingsbondcalc.htm.

   To compute the values for savings bonds, read the instructions on the page and click on Get Started. Fill in the information (you do not need to fill in the Bond Serial Number field) and click on Calculate.

WEB REFERENCES

http://www.bloomberg.com/markets/

Under Rates & Bonds, you can access information on key interest rates, U.S. Treasuries, government bonds, and municipal bonds.

http://www.teachmefinance.com

A review of the key financial concepts: time value of money, annuities, perpetuities, and so on.

WEB APPENDICES

Please visit the Companion Website at http://www.pearsonglobaleditions.com/Mishkin to read the Web appendix to Chapter 4.

Appendix 1: **Measuring Interest-Rate Risk: Duration**